Short-term Ambiguity Assessment to Augment Tracking Data Association Information

Sabino Gadaleta, Shawn Herman, Scott Miller,
Fritz Obermeyer, Benjamin J. Slocumb, and Aubrey B. Poore
Numerica Corporation,
P.O. Box 271246,
Fort Collins, CO 80527
(970) 419-8343
{smgadaleta,abpoore}@numerica.us

Mark Levedahl
Engineering Fellow
Raytheon Company
1801 Dumbarton St.
McLean, VA 22101
(703) 418-4274
mlevedahl@raytheon.com

***Abstract*— A tracking system performs both state estimation and data association. Most trackers provide as output tracks with kinematic uncertainty information but no measure concerning data association uncertainty. In scenarios with closely spaced objects, even the most advanced trackers will sometimes produce impure tracks. Functions that process tracks, however, often implicitly assume correct data association and may produce suboptimal results if the association uncertainty is ignored. This paper describes how to augment a multiple hypothesis tracking system with association uncertainty measures. We show how the assignment probabilities are computed from ranked association hypotheses. Based on simulations we illustrate the use of these "short-term" association uncertainty measures in identifying potentially false correlations. To improve tracking performance in the presence of ambiguity we propose to use entropy of the assignment problem to adjust the length of the sliding window. We then discuss an approach to maintain "long-term" association uncertainty, based on Bayesian Networks, that accumulates short-term uncertainty information provided from the multiple hypothesis tracker.**

I. Introduction

A central component to a surveillance system is the tracking system that generally serves two purposes. First, it solves the data association problem of correlating measurements between frames into tracks, such that the observations that form a track most likely belong to the same truth object. Second, it performs state estimation by computing an estimate of the track's position, velocity, and acceleration. Modern tracking systems typically use a *multiple hypothesis data association* framework that maintains multiple data association hypotheses within a sliding window [1]. Two popular approaches are Multiple Hypothesis Tracking (MHT) [1] and Multiple Frame Assignment (MFA) [2], [3], [4], [5] tracking.

It has long been recognized [1] that, in the presence of Closely-Spaced Objects (CSOs), tracking systems often suffer from frequent misassociations and produce *impure* tracks. While MHT and MFA tracking systems are far less susceptible to misassociations than single hypothesis tracking systems, incorrect associations still occur in dense environments due to *ambiguity* in the correlation process.

Nonetheless, tracking systems typically report tracks only with measures concerning the uncertainty in the state estimation process, e.g., position and velocity covariance, and provide no measure concerning the uncertainty in the data association process. Down-stream algorithms, e.g., Target ID algorithms, often implicitly rely on the assumption of perfect or near-perfect data association. Thus, it is important to augment tracking data with measures concerning the estimated certainty in the reported data association solutions. This information can then be used by down-stream algorithms to correctly incorporate the uncertainty in the data association process such that, e.g., more robust and more informed target identification decisions can be made. The objective of this paper is to present a framework to exploit multiple data association hypotheses, either within the MHT or MFA, to compute measures of *data association ambiguity*. This allows one to extract soft-association information from the multiple hard-assignment solutions. The ambiguity is extracted without much additional computational cost to the tracking system.

This paper is structured as follows: Section II discusses tracking uncertainty. Section III reviews multiple hypothesis target tracking. Section IV details how to extract measures of short-term ambiguity from an MHT/MFA tracker and discusses some applications. Specifically, Section IV-A describes the implementation of a k-good N-D solver, Section IV-B presents results demonstrating that the computed assignment probabilities identify potentially incorrect assignments, and Section IV-D discusses a simulation concerning the use of assignment entropy as a means of adjusting the length of the extension window. To maintain long-term uncertainty we are currently developing a Bayesian Network Tracking Database that relies on short-term ambiguity information received by the multiple hypothesis tracker. This is discussed in Section IV-E. Section V concludes the paper.

II. Types of Uncertainty in Tracking

In tracking one can generally distinguish between state estimation and data association uncertainty. State estimates are reported with covariances that represent a measure of the uncertainty that results from noisy measurements and target mismodelling. However, most reported covariances are optimistic due to the assumption of correct data association in the filtering procedure. Data association uncertainty represents the uncertainty in the data correlation process. Some specific sources for correlation uncertainty are: bias, crossing targets, merged measurements, clutter, and missed detection. We can distinguish between between *short-term* and *long-term* association uncertainty. Possible questions concerning short-term ambiguity are: Is a specific measurement-to-track correlation certain? Is a measurement that was not assigned truly a false alarm? Is a new track truly from a new target? Possible questions concerning long-term ambiguity are: Is the track label attached to measurements reliable over long-term, i.e., did a track swap occur. Is a new track a new target or a track break? When we drop a track, is the target really not observed anymore? Can we connect tracks that were missed over long-term and form a track genealogy?

Figure 1 illustrates some association ambiguity through a two boat target scenario. The targets are initially separated 2 km and maneuver to within 50 m distance at time 150 s. At 300 s the boats separate, never crossing track. We simulate a Surface Search Radar (SSR) sensor based on (X-band) parameters specified from the "surface surveillance radar product brochure" from Raytheon (http://www.raytheon.com/products/sps73/). Our sensor model includes merged measurement, however, the targets are far enough apart for the sensor to resolve the objects during the scenario. The scan rate is 2.5 s. We use a Swerling I type model for the RCS with an SNR based detection test. Thus, in some scans, a target report may be missing due to a weak signal. Figure 1 shows the measurements as diamonds.

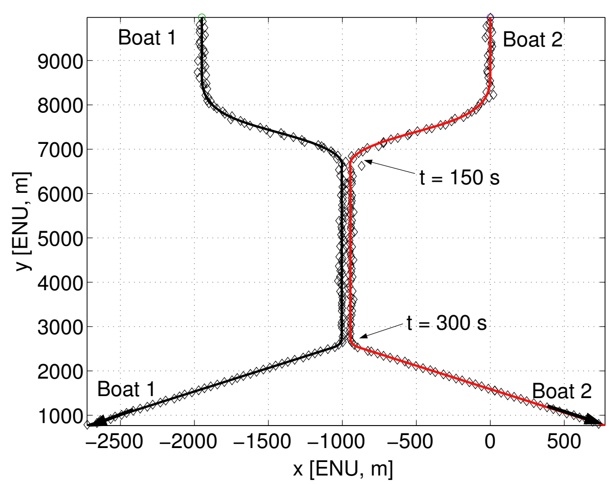

Fig. 1. Scenario simulating two boat targets observed by an SSR sensor.

Figure 2 shows simulation results obtained with an MFA tracker (using an IMM filter and a 10/2 window, see Section III). The IMM filter uses a low-noise and high-noise NCV filter and the filter is surface constrained using a pseudo-observation measurement update procedure [1]. The tracker produces three tracks during the scenario. Shortly after targets move into close vicinity, one track is discontinued and a new track initiated. We also notice that at the end of the scenario, the track that initially was on Boat 2 is now on Boat 1. Thus, several potential short-term and long-term uncertainties are present in the tracking data.

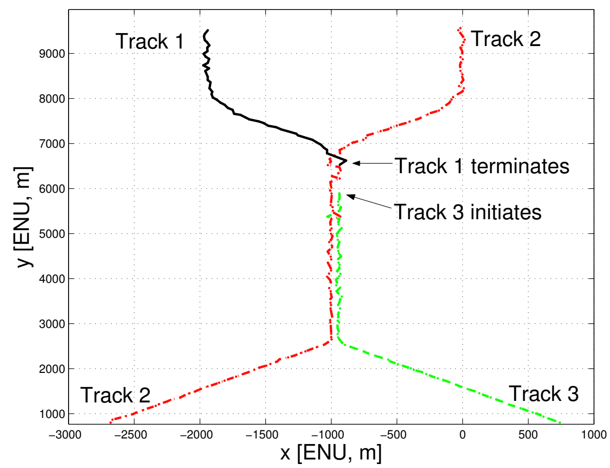

Fig. 2. Tracking results on two boat target scenario using a 10/2 sliding window.

The goal of this paper is to formulate a framework that allows one to extract short-term association ambiguity measures. We will present a simulation study that exploits an ambiguity measure to adjust the size of the sliding window. We will see that this can lead to improved tracking performance. We then briefly discuss a methodology that feed the short-term ambiguity information into a Bayesian Network Tracking Database (BNTD) to maintain long-term ambiguity measures.

III. Multiple Hypothesis and Multiple Frame Assignment Tracking

The central problem in multi-target, multi-sensor surveillance is the problem of partitioning measurement data into tracks, i.e., sets of observations that most likely originated from the same truth object, and false alarms, i.e., observations that most likely were produced by clutter. Let $\mathcal{Z}_n = \{\mathbf{z}^n_{i_n}\}_{i_n=1}^{m_n}$ denote a sequence of noise contaminated measurements produced by a sensor at time t_n. Let $\mathcal{Z}^M = \{\mathcal{Z}_n\}_{n=1}^{M}$ denote the data from M sensor scans, potentially from multiple sensors. The central data association problem in multi-target, multi-sensor data fusion can be generally posed as [5],

$$H^* = \arg \max_{H \in \mathcal{H}_M} \left\{ \frac{P(H|\mathcal{Z}^M)}{P(H_0|\mathcal{Z}^M)} \right\}, \tag{1}$$

where H denotes a partition of the data into tracks and false alarms, H_0 denotes a reference partition in which all reports are declared to be false alarms, $\mathcal{H}_M$ denotes the set of all feasible data partitions of the data $\mathcal{Z}^M$, and H^* denotes the optimal partition. Since the number of data partition hypotheses grows exponentially with the number of frames, the solution of the association problem requires an approximation. This leads to multiple frame processing, where data association hypotheses are maintained within a sliding window of $N \leq M$ data frames. To obtain the best data partitioning at the front of the window for the current frame, one needs to solve an $(N+1)$-dimensional assignment problem.

In *single-frame* tracking systems, association decisions are made immediately, based on the information from a single frame of data, i.e., $N = 1$. The single-frame approach requires the solution of a *two-dimensional assignment* problem and maintains only a single data association hypothesis. The most commonly used algorithm for tracking applications involving two-dimensional assignments is probably the adaptation of the Jonker-Volgernant algorithm by Drummond and Castañon [10], [11].

While 2-D assignment algorithms give optimal solutions, these solutions are not necessarily optimal for the tracking problem over more than two frames. Multiple frame methods are a computationally tractable approximation to the optimal solution (that would consider all data over all time). Two popular multi-frame tracking approaches can be distinguished: Multiple Hypothesis Tracking (MHT) [1] and Multiple Frame Assignment (MFA) [2], [3], [4], [5] tracking.

In the recommended MHT approach [1], the solution of the multi-dimensional assignment problem is approximated by using a sequential two-dimensional k-best assignment approach based on Murty's algorithm [12], [13]. At any time, the MHT maintains a set of k complete data association hypotheses H_i. Given a new frame, the H_i hypotheses are extended efficiently into k new hypotheses by a Murty-like algorithm [1]. Computational efficiency is obtained by maintaining hypotheses through a track-tree data structure [1].

The MFA approach solves the full multi-dimensional assignment problem through the use of an N-D assignment solver. This formulation is a superset of almost all MHT approaches to the multi-frame processing. Given N frames, the multi-dimensional assignment problem for tracking can be formulated compactly in the form [5]:

$$\begin{aligned}
&\text{Minimize} \sum_{i_1=0}^{M_1} \cdots \sum_{i_N=0}^{M_N} c_{i_1\cdots i_N} x_{i_1\cdots i_N} \\
&\text{Subject To:} \\
&\sum_{i_2=0}^{M_2} \cdots \sum_{i_N=0}^{M_N} x_{i_1\cdots i_N} = 1, \quad \text{for } i_1 = 1,\ldots,M_1, \\
&\sum_{i_1=0}^{M_1} \cdots \sum_{i_{k-1}=0}^{M_{k-1}} \sum_{i_{k+1}=0}^{M_{k+1}} \cdots \sum_{i_N=0}^{M_N} x_{i_1\cdots i_N} = 1, \\
&\quad \text{for } i_k = 1,\ldots,M_k \text{ and } k = 2,\ldots,N-1, \\
&\sum_{i_1=0}^{M_1} \cdots \sum_{i_{N-1}=0}^{M_{N-1}} x_{i_1\cdots i_N} = 1, \quad \text{for } i_N = 1,\ldots,M_N, \\
&x_{i_1\cdots i_N} \in \{0,1\} \quad \text{for all } i_1,\ldots,i_N,
\end{aligned} \tag{2}$$

where M_j denotes the number of objects in frame j. Here, $c_{0\cdots 0}$ is arbitrarily defined to be zero and is included for notational convenience. The zero index is used to represent missing data, false alarms, initiating tracks, and terminating tracks. We assume that the binary variables $x_{i_1\cdots i_N}$ with precisely one nonzero index are free to be assigned and that the corresponding cost coefficients are well-defined. The construction of real-time solutions of this fundamental problem has been achieved by the use of Lagrangian relaxation techniques [14].

The Numerica MFA tracking system maintains all association hypotheses in a moving window of frames and uses a dual-pane sliding window to maintain track hypotheses for both track initiation and track extension. We denote the size of the window by $N_{\text{init}}/N_{\text{ext}}$ with $N_{\text{init}} > N_{\text{ext}}$, and N_{init} denoting the number of frames used for track initiation and N_{ext} denoting the number of frames participating in track extension. All N_{init} frames within the window participate in track initiation but only the most recent N_{ext} frames participate in track extension (Figure 3 illustrates a 4/2 window). As the window moves, only the oldest frames of data are permanently assigned to tracks or false alarms. We refer to these permanent tracks as *firm*, or *established* tracks. The remaining assignments can be changed based on future frames of data.

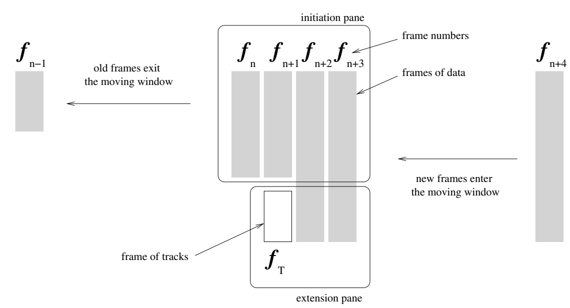

Fig. 3. Double pane moving window used in MFA tracking. The example shows a 4/2 window.

The formulation of the assignment problem in tracking algorithms (single-frame or multi-frame) requires the specification of the *likelihood ratio* of a partition hypothesis H,

$$L_H(M) := \frac{P(H|\mathcal{Z}^M)}{P(H_0|\mathcal{Z}^M)}, \tag{3}$$

which is equivalent to the product of the likelihood ratios of all tracks T in H: $L_H(M) \equiv \prod_{T\in H} L_T(M)$. Typically one uses a negative log-likelihood score instead: $c_T(M) := -\ln L_T(M)$, such that

$$c_H(M) := -\ln L_H(M) = \sum_{T\in H} c_T(M). \tag{4}$$

This implies

$$P(H|\mathcal{Z}^M) = e^{-c_H(M)} P(H_0|\mathcal{Z}^M), \tag{5}$$

which gives the *probability of a hypothesis* as a function of its cost and the probability of the null hypothesis. We will use this relation later to derive measurement-to-track data association probabilities. The likelihood ratio L_T of a track depends on a number of factors and its detailed formulation for measurement-to-track fusion is provided in [5]. Broadly, the track likelihood is composed of a kinematic, signal, and a feature component [1]:

$$L_T(M) \propto \prod_{n=1}^{M} L_{\text{kin}}(n) L_{\text{sig}}(n) L_{\text{fea}}(n). \tag{6}$$

The signal related term incorporates parameters such as a probability of detection or SNR and is described in detail in [5]. The feature component allows one to incorporate feature data, if available, to improve data association. The kinematic term requires a dynamics and measurement model and can be obtained through a filter algorithm, e.g., EKF or IMM, based on the innovation in the measurement-to-track update [1].

IV. Short-Term Ambiguity Assessment

An MHT system directly maintains a set $\mathcal{H}_M$ of k_M "complete" data association hypotheses $H_i \in \mathcal{H}_M,\ i = 1, \ldots, k_M$ at time t_M within a sliding window of sensor data frames to approximate the optimal solution that maximizes Eqn. (1). An MFA tracking system maintains an enumeration of track extension hypotheses within the window. Poore and Yan [15] previously introduced an algorithm that finds multiple quality solutions to the N-D assignment problem. Each alternative solution represents a different complete data association hypothesis H_i.

A. Finding k-Good Solutions to the N-D Assignment Problem

The partial branch-and-bound approach of Poore and Yan [15] for generating multiple good feasible solutions to Eqn. (2) is based on the Lagrangian relaxation method of Poore and Robertson [14] to find a single good solution. That algorithm may be divided into two steps. The first step is relaxation of the N-D assignment problem into a 2-D assignment problem using Lagrange multipliers to bring the final $N-2$ sets of constraints into the cost. For any given set of multipliers, the 2-D problem can be solved exactly using, for instance, an adaptation of the JV algorithm [10], [11] as discussed previously. Finding the best multipliers involves minimizing a nonsmooth convex function, which is achieved using a bundle method. The second step is a primal recovery phase. The optimal solution of the relaxed problem determines the assignments between the first two indices (which are feasible by construction), and what remains to be solved is an $(N-1)$-D assignment problem. The algorithm is called recursively, reducing the dimension by one each time, until the final primal recovery problem is 2-D and is solved exactly.

The partial branch-and-bound algorithm of [15] branches on alternative feasible solutions to the 2-D relaxed problem at each recursive stage, where alternatives are drawn from the m best points visited during the nonsmooth optimization, and "best" is judged by a combination of subgradient size and objective value. Here m is the branching number, which determines how many alternatives are explored at each branching step; typically $m = k$, but it can be different. The bound is obtained from the fact that the cost of any set of multipliers for a relaxed problem (a dual cost) is a lower bound on the costs of all the primal feasible points that can be recovered from it. Therefore, if the algorithm encounters any dual cost higher than a previously seen primal cost, that branch is pruned. The running k best primal solutions obtained from the final 2-D recovery phase may be kept for ambiguity assessment. Another branching method is to compute m-best solutions to each of the 2-D relaxed problems using Murty's method [12], [13]; the same bounding method applies. Neither of these approaches is guaranteed to find the k-best solutions to Eqn. (2), but they tend to generate k-good quality feasible solutions.

The k-good algorithm of Popp *et al.* [16] differs from the approach described above in several respects. It is based on the algorithm in [17] for finding a single solution to the N-D assignment problem. This method also relaxes the last $N-2$ sets of constraints to reduce the N-D problem to a 2-D problem, and searches for optimal Lagrange multipliers using a nonsmooth optimization technique. However, a primal feasible solution is recovered at *each step* of the nonsmooth optimization, in order to obtain an estimate of the duality gap for use in a termination criterion. The $(N-1)$-D primal recovery problem is solved approximately by a sequence of 2-D assignments, rather than a recursion of the algorithm. The use of a duality gap estimate can permit early termination with a provably good solution without obtaining optimal multipliers, but this comes at the cost of solving $(N-2)$ 2-D assignment problems at each nonsmooth iteration. In the method of [14] only one 2-D problem is solved per nonsmooth iteration, but it must solve $N-2$ nonsmooth problems of decreasing size. So if the same number of nonsmooth steps is performed everywhere, the two methods solve the same number of 2-D problems. However, one would expect the recovered primal solutions from [14] to be of higher quality than those obtained from a sequential 2-D approximation; otherwise, a sequential 2-D approximation would be sufficient for the original N-D problem.

The branching method in [16] is also different. A variation on Murty's method is employed, in which arcs of the full N-D problem are fixed or removed to partition the feasible solutions, and these N-D subproblems are re-solved to generate alternative solutions. The application of Murty's method on the 2-D relaxed problems as described above would appear to be more efficient, though, since new 2-D solutions can be generated very quickly from previous ones by a single augmentation [13].

B. Computing Assignment Probabilities

In the following, let $\mathcal{H}_M = \{H_1, H_2, \ldots, H_{k_M}\}$ denote a set of k_M *ranked* solutions to the assignment problem Eqn. (2) at time t_M, such that H_1 corresponds to the "best" solution returned from the assignment solver, H_2 to the "second-best", etc., i.e.,

$$c_{H_1}(M) \leq c_{H_2}(M) \leq \ldots \leq c_{H_{k_M}}(M), \tag{7}$$

noting that costs are defined as negative log-likelihood terms. As long as k_M is sufficiently large, it is the case that

$$P(H_0|\mathcal{Z}^M) + \sum_{i=1}^{k_M} P(H_i|\mathcal{Z}^M) \approx 1, \tag{8}$$

and therefore

$$P(H_i|\mathcal{Z}^M) = \frac{P(H_i|\mathcal{Z}^M)}{1} \tag{9}$$

$$\approx \frac{P(H_i|\mathcal{Z}^M)}{P(H_0|\mathcal{Z}^M) + \sum_i^{k_M} P(H_i|\mathcal{Z}^M)}. \tag{10}$$

We can now use Eqn. (5) to obtain [1], [16]

$$P(H_i|\mathcal{Z}^M) = \frac{e^{-c_{H_i}(M)}}{1 + \sum_i^{k_M} e^{-c_{H_i}(M)}}, \tag{11}$$

which gives the *probability of a hypothesis in terms of its cost* $c_H(M)$. Note that Eqn. (7) implies

$$P(H_1|\mathcal{Z}^M) \geq P(H_2|\mathcal{Z}^M) \geq \ldots \geq P(H_{k_M}|\mathcal{Z}^M). \tag{12}$$

As stated in [16], it may be necessary to "shift" the costs in Eqn. (5) for numerical stability:

$$P(H_i|\mathcal{Z}^M) = \frac{e^{-c_{H_i}(M)+c_s}}{e^{c_s} + \sum_i^{k_M} e^{-c_{H_i}(M)+c_s}}, \tag{13}$$

where a possible choice for the cost-shift is $c_s = c_{H_1}$.

The probabilities $P(H_i|\mathcal{Z}^M)$ of the multiple data association hypotheses are directly related to the probabilities of the data assignments, i.e., measurement-to-track correlations, as shown next. Let χ_{ijk} denote the *probabilistic data association event* that measurement $\mathbf{z}_j$ associates with system track T_i within sensor data frame $\mathcal{Z}_k$. Then, the probability $P(\chi_{ijk})$ of this event is given by

$$P(\chi_{ijk}) \approx \sum_{H \in \mathcal{H}_{ijk}} P(H), \tag{14}$$

where $\mathcal{H}_{ijk} \subseteq \mathcal{H}_M$ denotes the subset of those data association hypotheses from the set $\mathcal{H}_M$ that postulate the event χ_{ijk}. Thus, to compute the assignment probability $P(\chi_{ijk})$ we simply need to find all those data association hypotheses in which the measurement $\mathbf{z}_j$ is assigned to track T_i at frame $\mathcal{Z}_k$.

Tracking systems typically are tasked to report correlated measurements with estimates concerning track position and velocity. Most tracking systems report primarily measures concerning the uncertainty in the state estimate, i.e., position and velocity covariance. The assignment probabilities $P(\chi_{ijk})$ serve as an additional measure concerning the *data association uncertainty* or *ambiguity*. If $P(\chi_{ijk}) < 1$, the tracking system estimates that there is a non-zero probability that the (reported) measurement-to-track correlation from the best solution H_1 is incorrect. Some applications, e.g., Target ID, rely heavily on the assumption that data correlation is correct. If the assignment probabilities $P(\chi_{ijk})$ are available to this application, then those measurements that correspond to uncertain correlations can be treated properly (or ignored completely).

1) Simulation Study: We present a simulation study using the two boat scenario discussed in Section II. We use the Numerica MFA tracking system with a 10/2 sliding window. Exploiting k-good solutions to the multi-frame assignment problem we estimate the assignment probabilities $P(\chi_{ijk})$ of reported tracks. Figure 2 in Section II showed the tracking results obtained. Figure 4 shows the assignment probabilities for the three established tracks. Marked as "o" are assignments where the best solution postulates a measurement-to-track assignment. Marked as "*" are assignments where the best solution postulates a missed detection. Measurement-to-track assignments marked with a big dot correspond to best-solution-misassociations. We use measurements tagged with Truth IDs to identify misassociations and consider a best-solution-correlation as a misassociation if the previous correlation and the current correlation correspond to different truth targets. In this scenario we identified a total of 10 incorrect assignments. The results show that 8 of the 10 incorrect correlations correspond to low-probability correlations and are thus identified as potentially false by the tracking system.

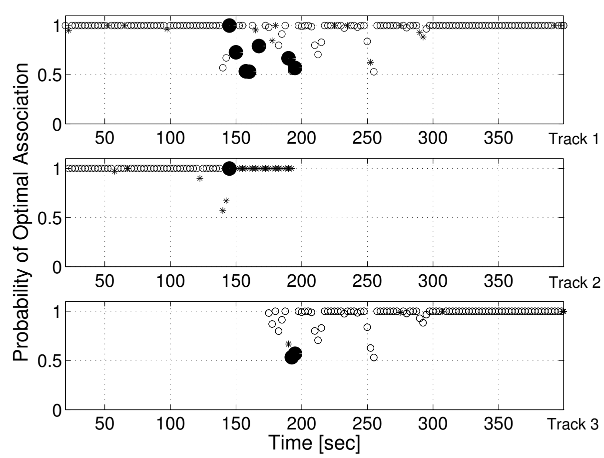

Fig. 4. Two boat scenario running an MFA tracker with a 10/2 window. The different panels show the best-solution assignment probability of the correlations over scenario time for (upper panel) Track 1, (middle panel) Track 2, and (lower panel) Track 3. See text for details.

Given the assignment probabilities, the output of the tracking can be *adapted* to an up-stream algorithm such that correlations χ_{ijk} that have a probability $P(\chi_{ijk})$ below a certain threshold δ_{th} are removed from the tracker's output. Using this capability, the purity of track correlations reported to, e.g., a Target ID algorithm may be significantly improved. The dashed line in Figure 5 shows the percentage of pairwise correlations that were incorrect (two consecutive correlations are incorrect if they correlate different truth objects to the system track) over the last 25 s of scenario time. Thus, during the target crossing about 5% of pair correlations are false. With ambiguity processing, those correlations could be removed that belong to uncertain or ambiguous correlations. The red curve in Figure 5 shows the percentage of certain but false pairwise correlations over scenario time where a pair correlation is certain if both correlations have a probability greater than 0.9. We see that the number of potential correlation errors drops significantly during the period where ambiguity exists, leading to tracks of improved purity as output of the tracking system.

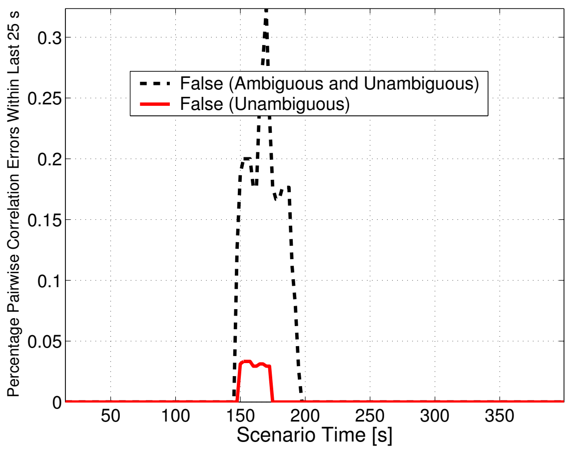

Fig. 5. Percentage of false pair correlations within the last 25 s of scenario time as a function of scenario time. The dashed line represents output without ambiguity processing and the red line represents output of the tracker if only certain correlations are reported.

C. Ambiguity Track Groups/CSO Clusters

A track, for which $P(\chi_{ijk}) < 1-\delta$, where δ is a threshold parameter, has non-negligible ambiguity in the correlation of measurement $\mathbf{z}_j$ to track T_i in frame $\mathcal{Z}_k$. This implies that at least one other track $T_{i'}$ is contending with track T_i about measurement $\mathbf{z}_j$ (this could be a new track hypothesis), i.e., a different data association hypothesis of non-negligible likelihood exists that postulates measurement $\mathbf{z}_j$ associating with track $T_{i'}$ instead of track T_i. Due to this ambiguity, the two tracks $T_{i'}$ and T_i are linked and may potentially represent the same target. Moreover, if track $T_{i'}$ and track T_i are linked by the common observation $\mathbf{z}_j$, and track $T_{i''}$ and T_i are linked by the common observation $\mathbf{z}_j$, then all three system tracks are in the same *ambiguity track group* or *Closely-Spaced Object (CSO) Cluster*. In general, a CSO cluster is defined as a group of tracks that are connected through ambiguous measurement-to-track associations. Future work will investigate the use of CSO clusters for merged measurement processing to extend the work of Gadaleta *et al.* [18]. The objective in this work is to correlate tracks within a CSO cluster with unresolved measurement data to seed a merged measurement signal-processing algorithm. As another future use, the information about CSO clusters may be useful to aid interceptor guidance since it may prevent redundant interceptor launch at potentially identical targets.

D. Adjusting Window Size According to Ambiguity

Given the probabilities $P(H_i|\mathcal{Z}^M)$, the *entropy* [19] of the assignment problem Eqn. (2) can be expressed in the form

$$\mathrm{entr}(\mathcal{H}_M|\mathcal{Z}^M) = -\sum_{H_i \in \mathcal{H}_M} P(H_i|\mathcal{Z}^M)\log_2 P(H_i|\mathcal{Z}^M) \tag{15}$$

in units of bits. The entropy can be interpreted as a measure of the uncertainty of the tracker's data correlation decisions. If the entropy $\mathrm{entr}(\mathcal{H}_M|\mathcal{Z}^M)$ is zero this implies that the data association decisions reported at time t_M were "certain" (as estimated by the tracker).* If the entropy is greater than zero, this implies ambiguity in the reported data associations. Thus, one potential approach to improve tracking performance in the presence of contention is to use measured entropy as a mechanism to adjust the size N_{ext} of the MFA tracker's extension pane. In future developments we envision using different window sizes on different *partitions* of the tracking problem. A partitioning may consist of sets of tracks such that no ambiguity between the tracks in different sets can exist. (Partitions can, e.g., be obtained through gating algorithms [1]). The objective then is to be able to use a larger N_{ext} only on tracks within ambiguous partitions. The entropy is then estimated separately for the respective partitions.

1) Algorithm to Use Entropy to Adjust N_{ext} Adaptively: An information measure related to entropy is *perplexity*. The perplexity is defined as $\mathrm{perp} = 2^{\mathrm{entr}}$. The perplexity $\mathrm{perp}(\mathcal{H}_M|\mathcal{Z}^M)$ can be interpreted as the average number of equally likely alternatives the information source (tracker) can choose from in specifying its message (data association solution). Given two frames of data, where Frame 1 contains n objects and Frame 2 contains m objects, assuming $n \geq m$, the number of possible assignment solutions to the two-dimensional assignment between Frame 1 and Frame 2 is given by $(n+1)!/(n+1-m)!$. Thus, maximum entropy for a two-dimensional assignment problem is:

$$\mathrm{entr}_{\max}(\text{2-D Assignment}) = \log_2\left(\frac{(n+1)!}{(n+1-m)!}\right). \tag{16}$$

To detect significant ambiguity within a partition, we propose to use a fraction, e.g., $\mathrm{entr}_{\text{th}} = 0.1\ \mathrm{entr}_{\max}$, and compare this threshold to a smoothed entropy estimate

$$\begin{aligned}\widehat{\mathrm{entr}}(\mathcal{H}_M|\mathcal{Z}^M) =& \alpha\ \mathrm{entr}(\mathcal{H}_{M-1}|\mathcal{Z}^{M-1}) \\ &+ (1-\alpha)\ \mathrm{entr}(\mathcal{H}_M|\mathcal{Z}^M),\end{aligned} \tag{17}$$

with forgetting factor $\alpha = 0.9$. If $\widehat{\mathrm{entr}} > \mathrm{entr}_{\text{th}}$, we increase N_{ext} by one (using four as an upper limit), otherwise we decrease it by one (using two as a lower limit). We use a smoothed entropy to avoid spurious adaptations of N_{ext} due to a fluctuating entropy.

*Note that a single-frame, single-hypothesis tracker would always estimate its association decisions to be certain since only a single hypothesis is formed. A single-frame tracker can be modified to perform ambiguity assessment if multiple hypotheses are formed for the two-frame assignment. Thus, generally a distinction between single-hypothesis and multiple hypothesis tracking is more useful than a distinction between single-frame and multi-frame tracking.

2) Simulation Study: Using the previously described two boat target scenario, we use entropy of the data association problem to adjust N_{ext}. We use $\text{entr}_{th} = 0.26$ as an entropy threshold (for $n = 2, m = 2$, it is $\text{entr}_{max} = 2.58$). Figure 6(a) shows the average entropy over time for the two boat scenario and Figure 6(b) shows the size of the extension pane following the described procedure in adjusting its size. We see that the window size is increased shortly before the two targets have fully moved into close formation. Shortly after the targets leave formation, the extension pane is reduced again to size two.

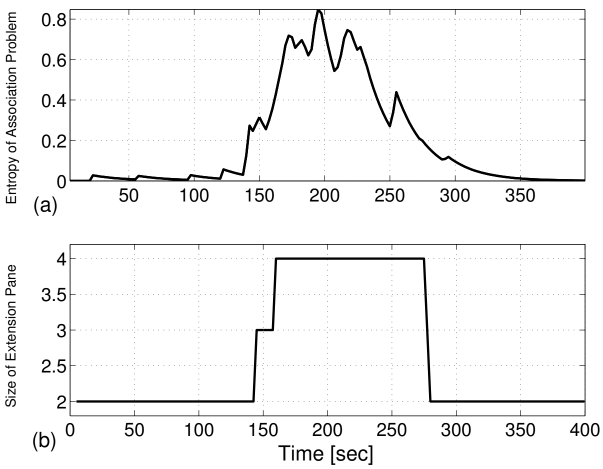

Fig. 6. Two boat scenario. (a) Smoothed entropy. (b) Size of the extension pane when using smoothed entropy to adjust the size of the window.

Figure 7 shows the tracks obtained with the adaptive window size tracking system. The tracking system now produces the correct number of tracks and results show that only 8 correlation errors are made. Thus, the tracking system using the adaptively sized window produces an improved tracking performance. However, we see that a track swap occurs: Track 1 is initiated on Boat 1 but is on Boat 2 at the end of the scenario. Thus, while the adaptive sliding window can improve tracking performance (especially in terms of the number of tracks produced), it may not be able to overcome long-term ambiguity when the ambiguity persists beyond the maximum window length.

Figure 8 shows the assignment probabilities for the two tracks with symbols as described previously for Figure 4. Track 1 shows no false correlations once the extension pane has expanded to size four. Track 2 shows four incorrect correlations after $t = 155$ s and those correspond to low-probability assignments.

E. BNTD Long-Term Uncertainty Management System

The k-Good approach for computing estimates of the data association ambiguity is being extended into a more complete *track uncertainty management system*. This system, called the Bayesian Network Tracking Database (BNTD) [20], works

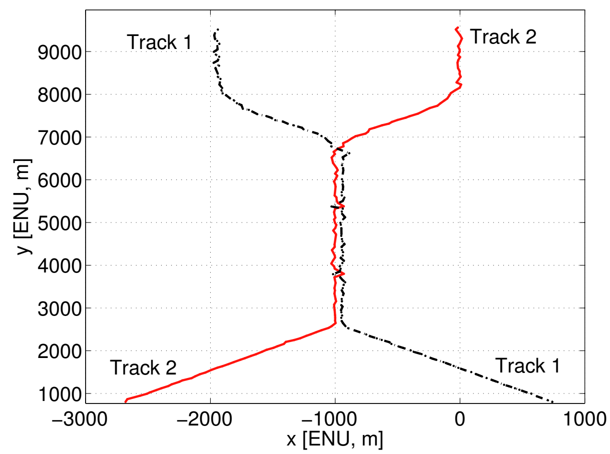

Fig. 7. Two boat scenario. Tracks obtained when using an MFA tracker with window size adapted according to entropy.

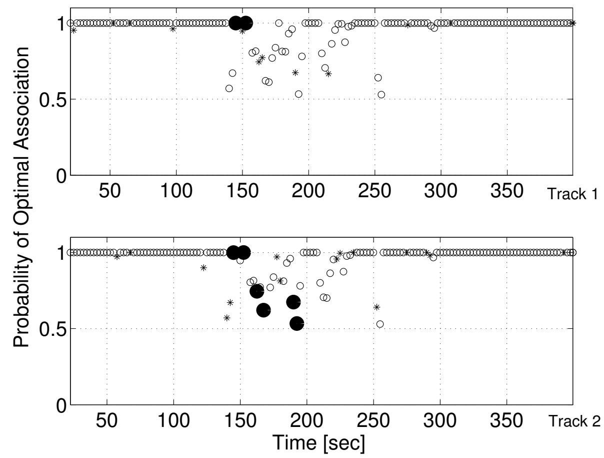

Fig. 8. Two boat scenario. Best-solution assignment probabilities when using an MFA tracker with window size adapted according to entropy.

along side a tracking system to capture the data association uncertainty information and then reports this information to an external system. The BNTD maintains data association uncertainty information by *accumulating the likelihoods of potential data associations* that are computed as tracks are formed. These likelihoods are provided by an MHT/MFA tracker and processed within the BNTD to generate short-term *and* long-term association uncertainty estimates, organized into a quickly accessible Bayesian network format (and not in a track file). Figure 9 illustrates the interaction between the MFA and BNTD. With the network, a system such as a classifier can investigate the ambiguity present in the tracks it is given, and consider the implications of other choices of data association, e.g., analyze the likelihood of a track swap.

The implementation of the BNTD system includes the following three main components: Storage and Accumulation, Network Propagation, and Query Response. The first com-

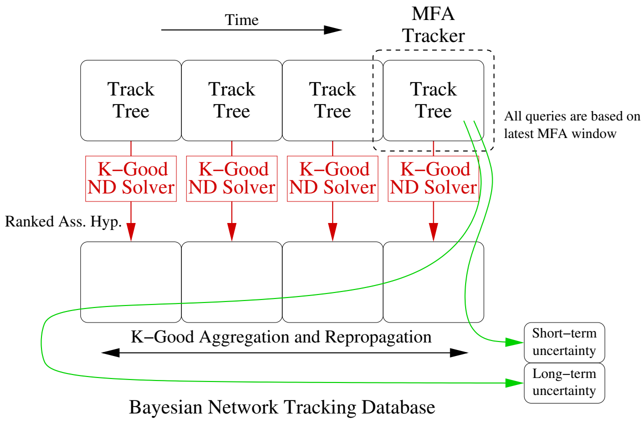

Fig. 9. Interaction between MFA tracker and BNTD.

ponent accumulates data association alternatives provided by the tracking system within a Bayesian network. The BNTD aggregator can process as input a set of k-good complete data association hypotheses provided by an MHT or MFA tracker, as described in Section IV. Such a set typically contains only recent short-term ambiguity information (based on the data within the MHT/MFA's sliding window). The BNTD aggregates these k-good sets into a coherent representation of long-term ambiguity, using efficient data structures. Once new data are provided to the network, the propagation component adjusts the probability flows (the node weights) through the network. The query response is formed by specifying prior evidence at certain nodes in the network to represent the query and then re-propagating the flows through the rest of the network. The BNTD can then report the flow values and "ambiguated" state estimates for any of the observations in the network in reply to the query.

An implementation of the BNTD has been developed, and refinements to the methodology are currently in progress. A future paper that details the technique is planned.

V. Conclusion

The purpose of this paper was to illustrate a framework that allows one to augment a multiple hypothesis tracking system with data association uncertainty measures. We described a k-good N-D solver that produces ranked quality solutions to the assignment problem formed within an MFA tracker. Based on these ranked solutions, assignment probabilities can be computed. These represent (short-term) association uncertainties and are valuable input to down-stream processing functions. We proposed to use entropy of the data association problem as a means to adjust the length of the sliding window in the multiple hypothesis tracker. While this may improve tracking performance, long-term ambiguities may remain. We briefly described a Bayesian Network Tracking Database (BNTD) that aggregates the short-term ambiguity information provided by a multiple hypothesis tracker to produce estimates of long-term ambiguity.

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